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# Comparisons and comments concerning recent calculations for flow past a circular cylinder

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This note has two aspects. The first concerns comparisons between the theoretical predictions of Smith (1979*b*) and the calculations of Fornberg (1980) for flow past a circular cylinder: there is quite good agreement overall, in such quantities as the drag, the front stagnation point pressure, the eddy pressure and the skin friction. The second aspect concerns comments and reservations on the calculated eddy lengths and discrepancies at higher Reynolds numbers.

## 1. Text

This note has two intentions: first, to compare the theoretical predictions of Smith (1979*b*) with the recent calculations of Fornberg (1980), referred to hereinafter as S and F, respectively; and secondly, to point out certain reservations concerning F's calculations. The notation of S is adopted here, with the Reynolds number $R(=2Re)$ based on the cylinder diameter.

Almost all aspects of F's results for a circular cylinder up to $R = 300$ are in line with the S theory for flow past a bluff body. These aspects include the eddy pressure, the front stagnation point pressure, the drag $C_D$ and the skin friction, as the comparisons in figures 1–4 show; similar agreement can be shown to hold for the entire surface pressure (S, figure 4). Thus, in particular, the last $C_D$ value, at $R = 300$, of F's table 6 falls exactly on S's curve (S, figure 11*b* or present figure 1). F's front stagnation point pressure of 0·51 (F, table 4) at $R = 300$ compares with S's prediction of 0·510 (equation (2.27) of S, and present figure 2). F's eddy, or rear stagnation point, pressures of 0·17 and 0·09 (F, table 4) at $R = 100$, 300 respectively compare with S's predictions of 0·194 and 0·112 respectively (S, figure 10; present figure 3): see also Dennis & Chang (1970).

On those scores then there is good, even encouraging, agreement. While, as has been noted (S), the theory has still to be completed, if it can be, with regard to the reattachment process, the agreements observed above add further weight to the findings of Smith (1979*a*), Smith & Duck (1980) and Smith & Daniels (1981), that is, that extended Kirchhoff theory provides a correct limiting solution for the grossly separated laminar flow of an incompressible fluid as $R \to \infty$. Their limiting studies do have firm theoretical grounding with regard to the reattachment process.

There is also not unreasonable agreement with F's results for the eddy length $l$ (S's figure 8; F's figure 17; present figure 5) except for one curious feature: F's results seem to show a rather excessive internal discrepancy, when tested between the crude and finer numerical grids (F figure 12), such that $l$ is given as approximately

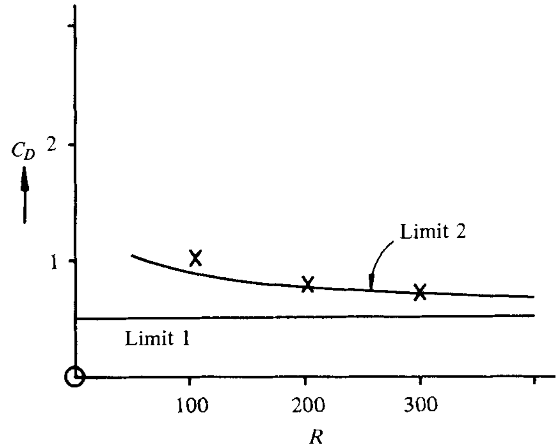


FIGURE 1. Comparison between the prediction in S (——) for $C_D$ (S, figure 11*b*) and F's numerical results (X), for $R$ up to 300. The leading-order prediction Limit 1 is the Kirchhoff limit $C_D = C_{D\infty} = 0{\cdot}50$, whereas Limit 2 is the prediction $C_D = 0{\cdot}50(1+7{\cdot}61\,R^{-\frac{1}{2}})$ including the higher order re-scaling effect of the eddy pressure and the friction drag. An $O(R^{-\frac{1}{16}})$ correction term is omitted since it is tiny in practice (S).

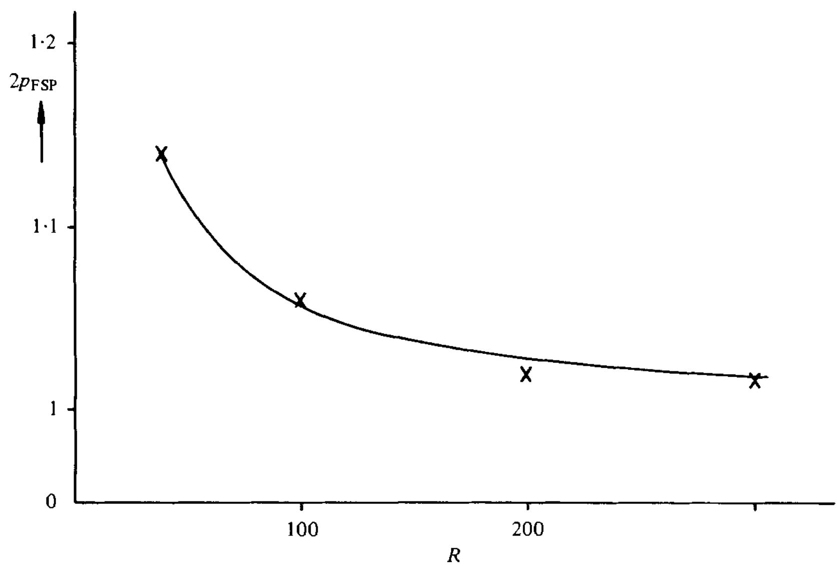


FIGURE 2. Comparison between the prediction in S (——) for the front stagnation point pressure $p_{FSP}$ (S, figure 2) and F's numerical results (X) for $R$ up to 300. The prediction (——) is $2p_{FSP} - 1 = 5{\cdot}692/R$ to leading order.

32·9 on one grid in F figure 12 (*e*) and as approximately 27·5 on the other in F figure 12 (*f*), even at $R = 200$. The discrepancy is therefore greater than 16% then. No corresponding tests are shown by F for $R > 200$ (only a minor test is applied then) and indeed only one set of results for $l$ is shown in F figure 17 for all $R$. We show more in the present figure 5, which indicates a fairly favourable comparison overall with the theory in S. It would be interesting to know, however, whether a possible decrease of $l$ for $R > 290$ (approximately) does occur, as the numerical results so far

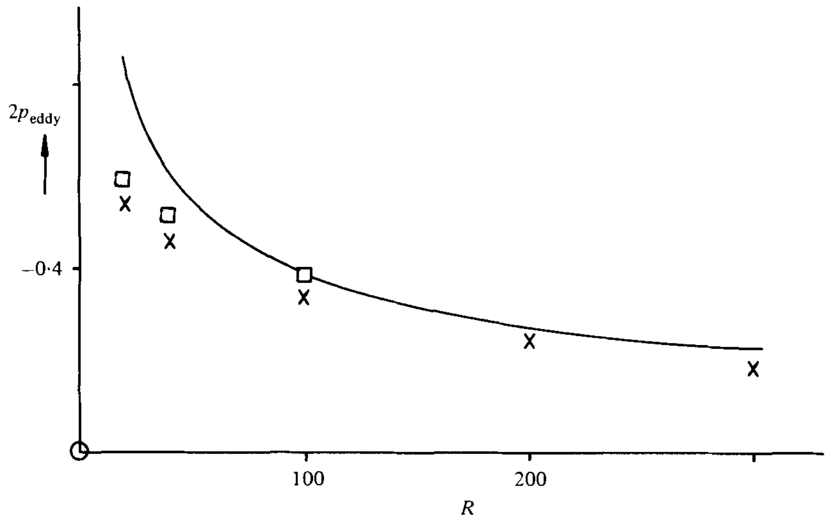


FIGURE 3. Comparison between the prediction in S (——) for the eddy pressure (S, figure 10) and F's numerical results (X), for R up to 300. Also shown are results from Dennis & Chang (1970) (□). The prediction (——) is from S's (3.2*b*), (3·16), with $C_{D\infty} = 0{\cdot}50$, $Re = \frac{1}{2}R$, and is $2\,p_{eddy} = -3{\cdot}88\,R^{-\frac{1}{2}}$ to leading order.

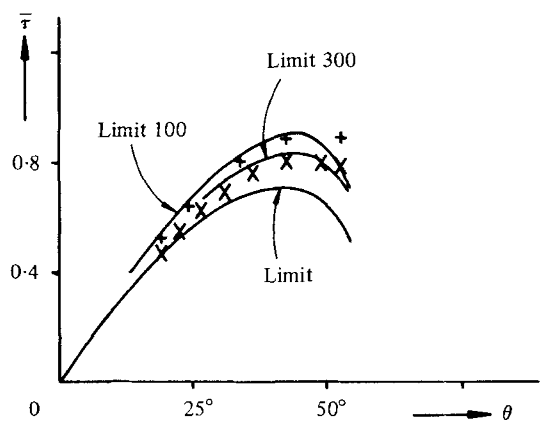


FIGURE 4. Comparison between the prediction in S (——) (S, figure 3) for the skin friction $\bar{\tau}$ versus distance $\theta$ from the front stagnation point and F's numerical results +, X (F, figure 15), for $R = 100$, 300 respectively. The prediction Limit is for the limit $R \to \infty$, whereas the predictions Limit 100, Limit 300 for $R = 100$, 300 include the higher order but important effects of the eddy pressure.

might be taken to suggest, or if it is merely a numerical rather than a genuine feature. For, according to the 16% or more numerical discrepancy at $R = 200$, the eddy length is clearly a very sensitive quantity to determine computationally (see also next paragraph) and is the only main one to exhibit any significant deviations from the predicted trend of S, and just at the very largest values of $R$ attainable in F's computations. Again, there is no corresponding deviation in $C_D$ (figure 1) which would seem necessary (S) for a theory incorporating a shorter eddy length for $R \to \infty$, while the behaviour of the eddy vorticity emphasized by F is not inconsistent with the S theory, given the apparent sensitivity of the eddy-length results. In view of all this and especially the 16% numerical discrepancy noted above we feel that, at least

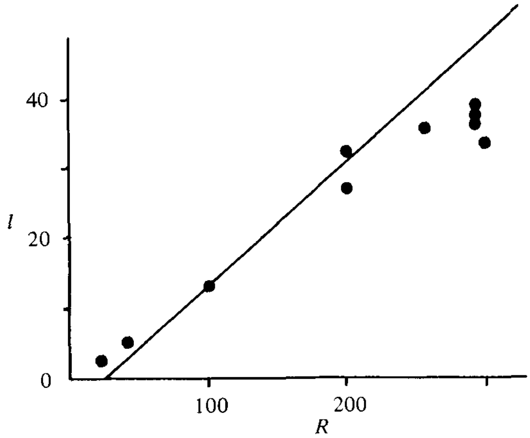


FIGURE 5. Comparison between the prediction in S (——) for the eddy length $l$ (S, figure 8) and F's various numerical results (●) (F, figure 12, 13*a–c*, 17) for $R$ up to 300. The prediction —— has slope $dl/dR = 0{\cdot}17$ to leading order.

until the calculations have been systematically checked, reservations about F's results for the eddy length, if nothing else, are inevitable.

For the purposes of comparison we observe the effects of systematic checking, on refined grids, in the computational work by Dennis & Smith (1980) on separating flow. Their table 1 (their p. 404) gives the eddy length $L_1$ versus Reynolds number $R$, calculated on *four* increasingly refined grids, in the flow ahead of a forward-facing step in a channel. The grid refinement becomes ever more necessary as $R$ increases and only with the refinement applied can $L_1$ be determined accurately. The accurate values of $L_1$ then agree well with the asymptotic theory, as their figure 2 shows, whereas the agreement diminishes significantly if the values obtained on cruder grids are taken instead. The analogy and contrasts between their sets of results and F's, the latter using essentially just two grids for $R \leqslant 200$ and only one grid for $R > 200$, and the resultant comparisons with asymptotic theory, are rather striking.

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